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RESEARCH AND ANALYSIS OF MULTIFRACTAL CHARACTERISTICS OF CRYPTOCURRENCY MARKETS

This study presents a multifractal analysis of the Bitcoin price time series over the period of 2015 to 2024. The multifractal fluctuation analysis with detrending (MFDFA) method is widely used to study fractal properties in financial time series.

The results of the MFDFA indicate that the multifractal spectrum of the Bitcoin price time series has a positive slope. The multifractal spectrum demonstrated greater volatility at small time intervals and more predictable behavior at large. The Hurst exponent, which is a measure of the long-term memory of the time series, is found to be 0.5191. This implies that Bitcoin have weak autocorrelation and little tendency to trend.

The results of the study provide new insights into the complexity of the Bitcoin market and contribute to the ongoing debate on the market efficiency of cryptocurrencies.

Keywords: Fractal, Financial Time Series, Multifractal Analysis, Bitcoin, Hurst Exponent.

Introduction

In recent years, the growth of cryptocurrencies has been an area of significant interest in finance and economics. Bitcoin, as the first and most widely used cryptocurrency, has been the subject of numerous studies aimed at understanding its price dynamics. Bitcoin is a decentralized digital currency that operates on a peer-to-peer network and allows for instant, low-cost transactions. Since its introduction in 2009, Bitcoin has gained widespread popularity and is widely considered as one of the most influential technological innovations of recent times. Despite its popularity and widespread usage, the underlying mechanisms that govern the price dynamics of Bitcoin remain largely unknown. In this study, we use multifractal detrended fluctuation analysis (MFDFA) [1] to shed light on the complexity of Bitcoin price movements and to better understand the underlying market dynamics.

The object of research is the financial time series.

The subject of the research is software and algorithms for analyzing the price of Bitcoin using the multifractal detrended fluctuation analysis.

The objective of the research is to investigate the fractal properties of the time series and their implications for market efficiency and volatility.

Data and Methodology

The data used in this study consists of minutely close prices of Bitcoin, spanning from 2015 to 2024 (fig. 1). The data was obtained from a publicly available source (https://www.cryptodatadownload.com) and contains more than 4.5 million records.

MATLAB version 9.8 was used to analyze and process the collected data. This software allows you to perform various calculations operating on large volumes of data, which is a necessary condition for the implementation of time series analysis algorithms.

Multifractal analysis is a technique for characterizing the statistical properties of financial time series data. The MFDFA has been applied in various financial time series, including stock returns, interest rates, and currency exchange rates [4]. It is based on the detrended fluctuation analysis (DFA) method, which is a non-parametric technique for characterizing long-range correlations in time series data. The MFDFA extends the DFA method [1] to provide a more comprehensive analysis of the multifractal nature of time series data [2]. One of the main benefits of multifractal analysis is that it can reveal the presence of fractal (self-similar) patterns in a time series that would be difficult to detect using other methods [6].

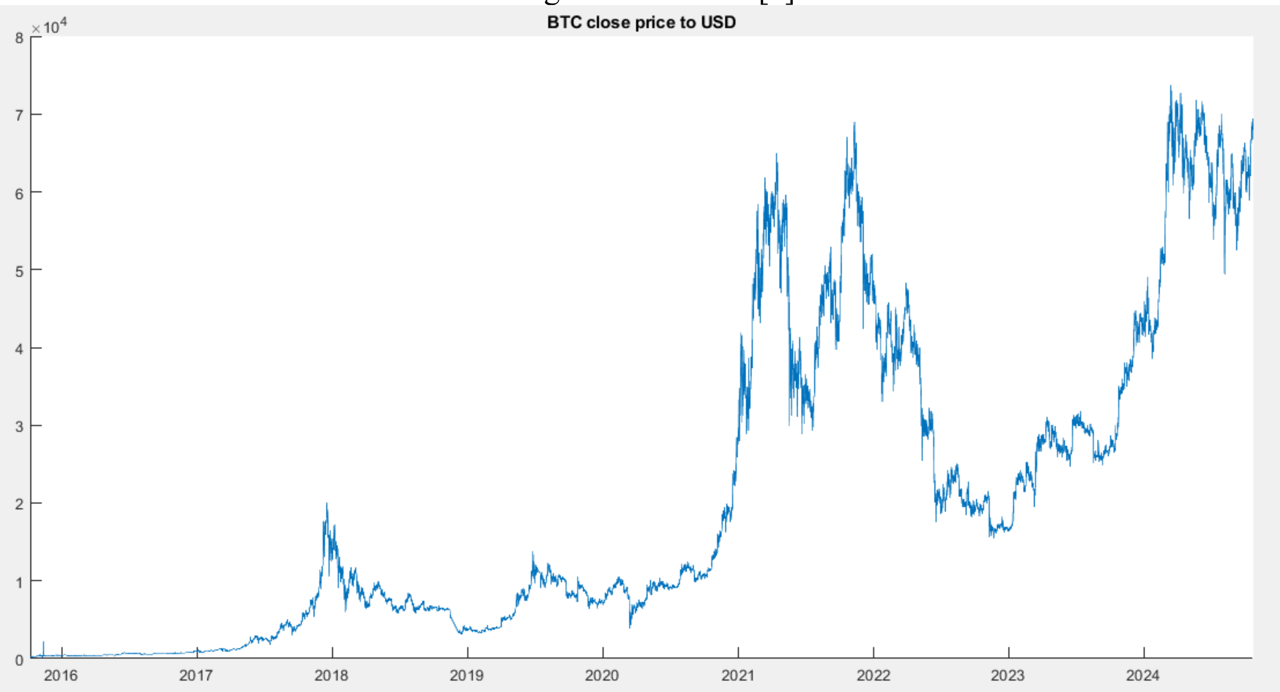

Figure 1. Bitcoin price chart

The basic idea of multifractal analysis is to partition the data into smaller segments and study the scaling behavior of the fluctuations at different scales [3]. To perform the MFDFA, we need to calculate the log returns of the close prices and analyze the profiles at various scales which are defined as shown in Table 1.

Table 1

Scales and number of time series points

10 minutes	15 minutes	30 minutes	1 hour	2 hours	4 hours	6 hours	8 hours	12 hours	1 day	2 days	3 days	5 days	1 week	1 month
10	15	30	60	120	240	360	480	720	1440	2880	4320	7200	10080	43200

The MFDFA algorithm then calculates the variance of profiles, which are created by dividing the log returns data into equal segments. The length of each segment is determined by the scale value. The variance of the profiles is then averaged over all segments to obtain the MFDFA value for a given scale. The process is repeated for all scales, and the MFDFA values are plotted against the scales on a log-log scale.

The process of Multifractal Detrended Fluctuation Analysis involves analyzing a timeseries by generating a fluctuation function $F^2(q,s)$ where q represents the q-powers and s represents the segment size. To perform MFDFA on a timeseries X_t, one needs to calculate the integral of the series as $Y_t = cumsum(X_t)$ and then segment it into N_s segments of size s (1).

$$F^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}[Y_{(v-1)s+i} - y_{v,i}]^2, for\ v = 1,2,\ldots,N_s \tag{1}$$

By performing polynomial fittings of order m on each detrended segment using *yv,i,* one can obtain the variances for each segment. These variances are then averaged over different segment sizes, starting with a small size s and gradually increasing it. This process leads to the generation of the fluctuation function (2) which is dependent on the segment length.

$$F_q^s(s) = \left\{\frac{1}{N_s}\sum_{v=1}^{N_s}[F^2(v,s)]^{q/2}\right\}^{1/q} \tag{2}$$

Once the fluctuation function has been generated, it can be plotted on a log-log scale [5]. The slope of the resulting plot, which represents the relationship between the fluctuation function and the s-segment size, is known as the self-similarity scaling *h(q) (3)*.

$$F_q^2(s) \sim s^{h(q)} \tag{3}$$

The multifractal spectrum shows the scaling behavior of the fluctuations of the Bitcoin price returns at different scales. If the curve has a negative slope, it suggests that the time series has persistent or long-memory behavior, meaning that large fluctuations are more likely to be followed by large fluctuations. If the curve has a positive slope, it suggests that the time series has an anti-persistent or short-memory behavior, meaning that large fluctuations are more likely to be followed by small fluctuations. The slope of a line can be calculated using the formula:

$$slope = \frac{(y_2 - y_1)}{(x_2 - x_1)} \tag{4}$$

where (x_1, y_1) and (x_2, y_2) are any two points on the line. If the resulting value is positive, then the slope is positive. If the resulting value is negative, then the slope is negative.

MFDFA is used to determine the Hurst exponent and to analyze the singularity spectrum of a time series [8]. The Hurst exponent is a commonly used measure of the long-term memory of the time series and can be used to determine whether the time series is persistent ($H > 0.5$), antipersistent ($H < 0.5$) or random walk ($H = 0.5$) and provides information about the underlying market dynamics [7]. A persistent time series exhibits positive autocorrelation, which means that the values are correlated over a long period of time. The Hurst exponent is calculated by fitting a line to the log-log plot of MFDFA values against scales and finding the slope of the line.

Results and Discussion

The results of the MFDFA reveal several interesting patterns in the price dynamics of Bitcoin. The multifractal spectrum curve has a positive slope, indicating that the Bitcoin price series is multifractal in nature (fig 2). This result is consistent with previous studies that have used MFDFA to analyze financial time series and suggests that the price dynamics of Bitcoin are characterized by the presence of multiple time scales.

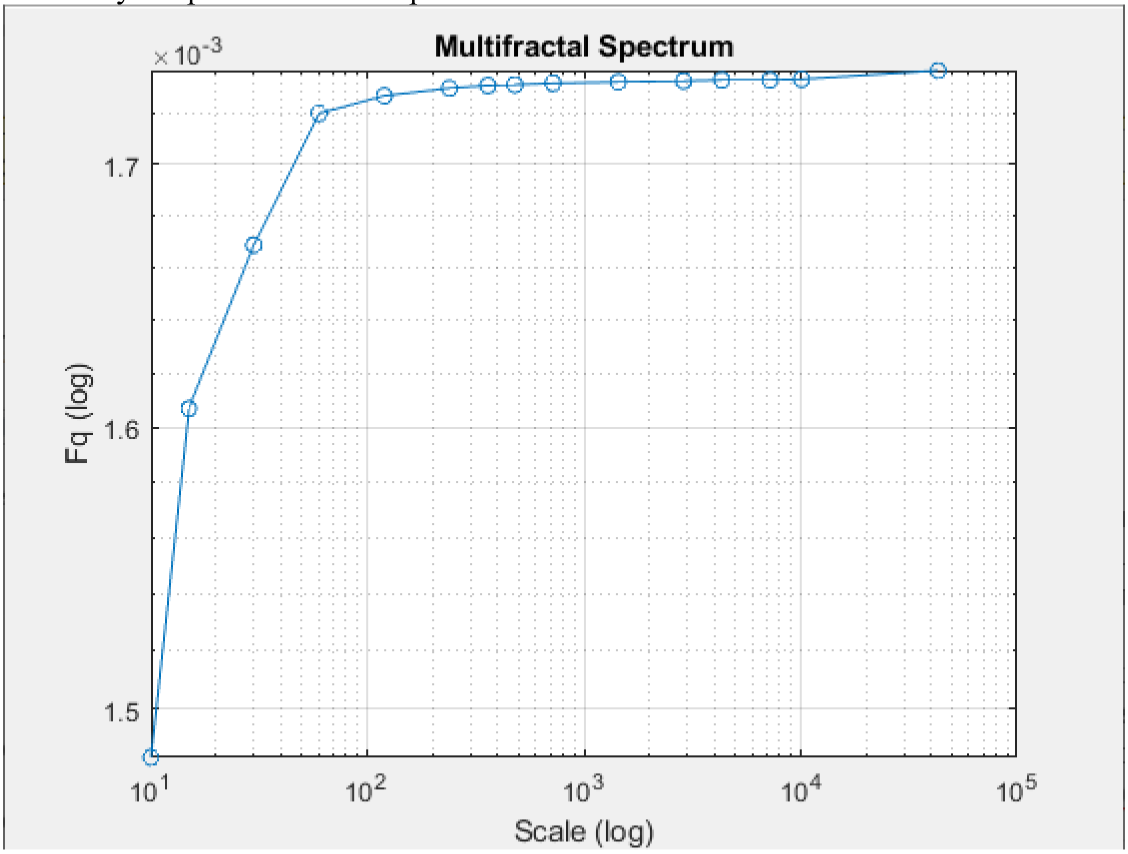

Figure 2. The multifractal spectrum chart.

The value of the Hurst indicator is $H = 0.5191$, which indicates the almost random nature of the bitcoin price movement in the analyzed time interval. This value indicates that the market has almost no strong trends or reversals. This is a typical result for many financial markets (Brownian motion). This result is consistent with previous studies that used MF-DFA to analyze financial time series.

The results of this analysis can be useful for financial traders and investors in understanding the self-affine properties of the Bitcoin market and making informed investment decisions.

It is important to note that the MFDFA is a powerful tool for analyzing the fractal properties of the time series, but it is subject to certain limitations and assumptions. For example, the MFDFA assumes that the time series is stationary and self-affine, which may not be the case for certain types of financial time series. It is also important to consider the effect of outliers and non-stationarity on the results of the MFDFA. Also, it is important to note that the results of this analysis are based on a specific dataset and may not hold for other financial markets or time periods. Further research is needed to confirm the generalizability of these results and to gain a deeper understanding of the underlying mechanisms that drive the multifractal structure of financial markets.

In future research, it would be interesting to explore the fractal behavior of other cryptocurrencies and to compare the results to those of Bitcoin. It would also be valuable to extend this analysis to include other financial time series, such as stock prices and exchange rates, to understand the generalizability of the findings. Additionally, it would be interesting to explore the relationships between the multifractal properties of different financial time series and their return and risk characteristics.

Conclusions

This study provides a comprehensive analysis of the complexity of Bitcoin price dynamics using MFDFA. The results suggest that the price movements of Bitcoin are multifractal in nature and that they can be effectively modeled as a singularity process. These findings provide valuable insights into the underlying market dynamics of Bitcoin and may be useful for traders, investors, and policymakers in making informed decisions.

Overall, this study highlights the importance of using advanced techniques such as MFDFA in analyzing complex financial time series and provides a valuable contribution to the literature on Bitcoin price dynamics because results of this study can be used as a reference for future studies of the multifractal properties of cryptocurrencies.

By providing insights into the distribution of fluctuations and singularities, multifractal analysis can help researchers better understand the underlying mechanisms driving these systems and develop more accurate predictive models.

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