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ENHANCEMENT OF MEDICAL MRI IMAGES BASED ON FRACTAL OPERATORS

Abstract. This paper presents texture enhancement algorithms for medical images. MRI brain scans often contain large, low-contrast grey areas that hold important information for doctors. Improving the texture helps highlight these areas for more detailed analysis. The study found that fractal operators are especially effective for processing medical images. The mathematical basis of these operators is explained through the Grünwald-Letnikov fractional derivatives. The process of creating fractal differential masks and applying them to enhance images is described. The fractal-based approach improves image qualities such as contrast, correlation, energy, and homogeneity. It was concluded that this method is well-suited for MRI image enhancement, both visually and based on metrics, making it a practical solution for real-world applications.

Keywords: Medical images, Magnetic Resonance Imaging (MRI), fractal operators, algorithms, image enhancement.

Introduction

Medical images are vital in modern medicine, helping doctors visualize body structures and diagnose diseases more accurately. Magnetic Resonance Imaging (MRI) provides detailed images of organs and tissues, but artifacts and noise can affect their quality. While equipment improvements and patient stabilization during scanning can reduce these issues, computer algorithms are, often more effective in enhancing image quality, especially when dealing with various resolutions, noise, and blurred contours [1].

The object of the study is the textures of medical MRI images.

The subject of the study is the methods and algorithms for improving MRI image textures using fractional derivatives.

The goal of the work is to apply mathematical tools, develop algorithmic solutions for improving medical MRI image textures using fractional derivatives.

Most existing texture enhancement methods are based on integer-order derivatives, including algorithms like the Laplacian [2], Sobel [3], and Pruitt [4] filters. These methods use classical differential operators to enhance edges and contours, but they have limitations: insufficient flexibility for details of different scales, potential ineffectiveness with thin or complex structures, and orientation sensitivity. MRI images, characterized by small, thin structures and self-similarity, require more adaptable approaches. Fractional differential operators, which generalize classical operators to non-integer orders, offer greater control over enhancing different frequency components, improving textures, and highlighting details. This article applies fractional derivatives to enhance image quality and contrast.

The fractial differential mask preserves low frequencies in smooth areas and enhances high frequencies where intensity changes, improving texture details. However, since noise is also high frequency, applying such filters can increase noise along with useful elements. The method by Yui-Fei, Pu, and others [5], based on fractional-order derivatives (YuiFeiPU operators), solves this problem. They propose constructing masks using Grünwald-Letnikov (YuiFeiPU2) derivatives. However, no studies compare fractal operators with other algorithms for MRI image enhancement. This work focuses on using Grünwald-Letnikov fractal operators to process MRI images.

Main material

There are several ways to determine the derivative of fractional order. Common generalized derivatives include the Riemann-Liouville, Grünwald-Letnikov, and Riesz derivatives.

Fractional derivative of order v based on Grünwald-Letnikov fractional differential is expressed by the formula

$$\mathrm{D}^{\mathrm{V}}_{\mathrm{G\text{-}L}}S(x)=\frac{d^{v}}{[d(x-a)]^{v}}s(x)_{G-L}=\lim_{N\to\infty}\left\{\frac{\frac{x-a}{N}}{\Gamma(-v)}\sum_{k=0}^{N-1}\frac{\Gamma(k-v)}{\Gamma(k+1)}*s\left(x-k\left(\frac{x-a}{N}\right)\right)\right\}, \tag{1}$$

Based on the above definitions, the main steps for applying the fractional operator to construct fractional differential masks, as described by the authors in [5], and the algorithm for their use are presented.

According to formulas from [5] mask coefficients are calculated using the formulas:

$$\begin{cases} C_{s_{-1}}=\dfrac{v}{4}+\dfrac{v^{2}}{8} \\ C_{s_{0}}=1-\dfrac{v^{2}}{2}+\dfrac{v^{3}}{8} \\ C_{s_{k}}=\dfrac{1}{\Gamma(-v)}\left[\dfrac{\Gamma(k-v+1)}{(k+1)!}\left(\dfrac{v}{4}+\dfrac{v^{2}}{8}\right)+\dfrac{\Gamma(k-v)}{k!}\left(1-\dfrac{v^{2}}{4}\right)+\dfrac{\Gamma(k-v-1)}{(k-1)!}\left(-\dfrac{v}{4}+\dfrac{v^{2}}{8}\right)\right] \end{cases}, \tag{2}$$

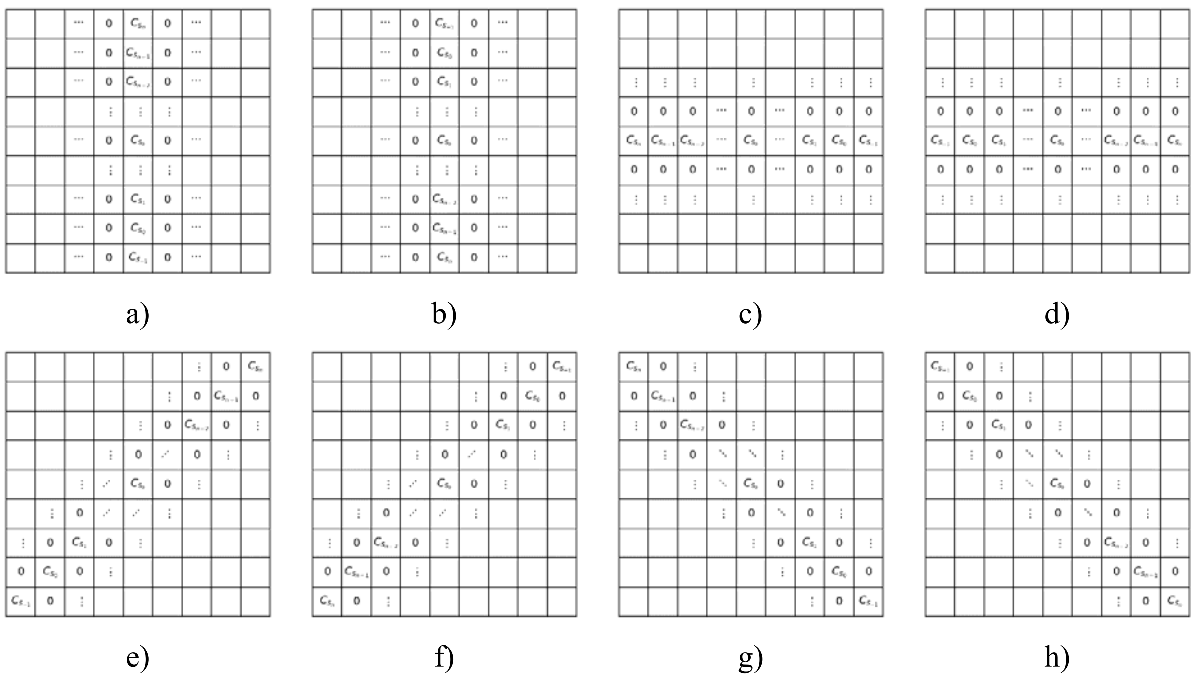

Figure 1: Маски для восьми напрямків (a) W1 від'ємній по осі x; (b) W2 додатній по осі x; (c) W3 від'ємній по осі y; (d) W4 позитивний по осі y; (e) W5 ліво висхідна діагональ; (f) W6 право спадна діагональ; (g) W7 право висхідна діагональ; (h) W8 ліво спадна діагональ [5]

In Fig.1, masks corresponding to the eight symmetrical directions are shown(a) W1 negative along the x-axis; (b) W2 positive along the x-axis; (c) W3 negative along the y-axis; (d) W4 positive along the y-axis; (e) W5 left-upward diagonal; (f) W6 right-downward diagonal; (g) W7 right-upward diagonal; (h) W8 left-downward diagonal.

These masks allow calculating the fractional differential in eight symmetrical directions.

Applying the software implementation of the algorithm for improving medical images for brain MRI, in the interval from v[0.5;0.9] with a mask of size 7x7, improved MRI images were obtained Fig. 2. (b), (c), (d), (e), (f). The images shows that the value of the order of differentiation v acts as the degree of image improvement, the more the value of v increases, the more gray colors disappear from the image, and as a result, the actual loss of gray areas of the image is obtained, while the edge areas become more visible. For the value v=0.5, on the contrary, the gray areas are strengthened, they became more visible, at the same time, the edges of the areas are quite strongly defined.

According to the visual comparison, it can be concluded that the fractal differential mask is suitable for improving images. Changing the value of v allows changing the degree of visibility of certain areas. Additionally, it is also observed that neighboring levels of order v look very similar, which means that there is a continuous interpolation of the fractional differential for derivatives of neighboring orders.

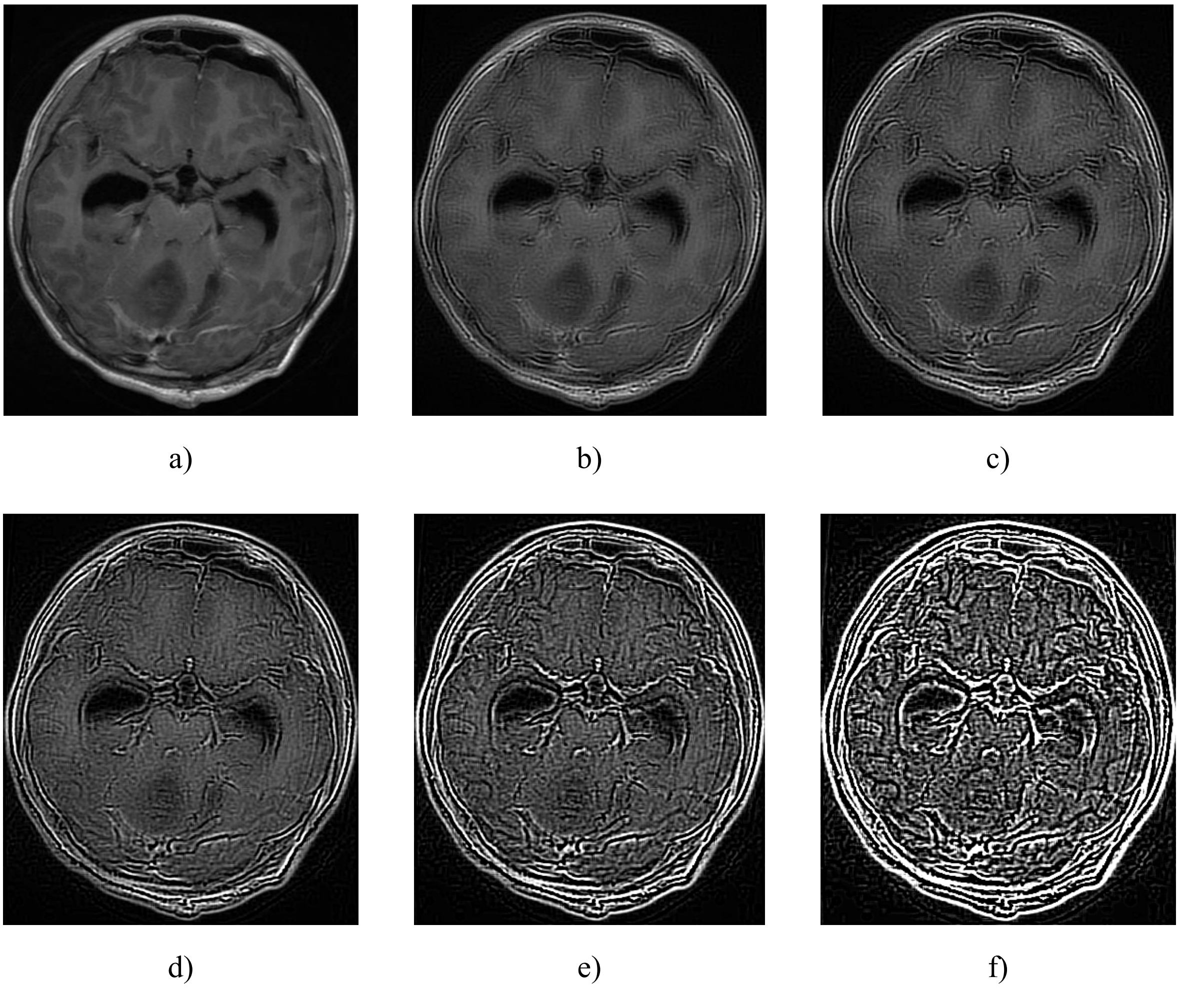

Fig. 2. Brain MRI (a) original image, (b) improved by fractional derivative of order 0.5, (c) improved by fractional derivative of order 0.6, (d) fractal derivative of order 0.7, (e) fractal derivative of order 0.8, (f) fractal derivative of order 0.9

Conclusions

Based on the results of this research, which included a review of the mathematical framework, algorithm development, and software implementation for enhancing the textures of medical MRI images using the fractal derivative, the following conclusions can be made:

The developed algorithm for mask construction, coefficient calculation, and mask application proved to be effective. The use of eight masks allowed for image enhancement regardless of the element's position in the image. The chosen mask size of 7x7 was optimal, ensuring a low calculation error.

These results confirm the viability of using the fractal derivative to enhance the textures of medical MRI images, particularly when detailed structure visualization is required. Therefore, the use of the fractal derivative represents a promising area for further research and development in medical image processing to improve image quality and analysis.

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