СЕКЦІЯ «МАТЕМАТИЧНЕ І ПРОГРАМНЕ ЗАБЕЗПЕЧЕННЯ»

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Reducing noise in medical MRI images using the Atangana-Baleanu fractional operator

Abstract. This work examines fractal differential operators and their usage in processing medical MRI images, focusing on the Atangana-Baleanu fractial operator. Numerical approximations of this operator are reviewed and describes how the approximation coefficients are calculated. Using these approximations, directionally oriented masks are created for image denoising, that are integrated into a custom algorithm and software to enhance image quality. A visual and analytical comparison is made between denoised and original images, and the results are compared against both fractal-based and traditional denoising methods. The study concludes with insights into the effectiveness of the Atangana-Baleanu fractal operator for MRI image processing.

Keywords – magnetic resonance imaging (MRI), Atangana-Baleanu fractional differential, algorithm, software.

This work explores the use of the Atangana-Baleanu fractal operator to enhance and denoise brain MRI images, emphasizing its potential to preserve essential structures and enhance diagnostic clarity. Fractal operators are used in image processing for tasks such as: noise reduction [1] and edge detection [2], providing advantages for modeling complex dynamics and memory effects. Unlike traditional filters, the Atangana-Baleanu operator accounts for system memory, providing smoother, detailed image enhancement.

$$ {}^{ABC}D_{a}^{\sigma} f(t) = \frac{M(\sigma)}{1-\sigma} \int_{a}^{t} f^{'}(\tau) E_{\sigma}\left(-\frac{(t-\tau)}{1-\sigma}\right) d\tau \qquad (1) $$

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${}^{ABC}{}_{\mathrm{a}}D^{\sigma} f(t)$-the Atangana-Baleanu fractional derivative of a function f(t) of order σ, M(·) – the normalization constant, E_{σ}-the generalized gamma function of Mittag-Leffler, f′(τ) – the first derivative of a function f(t), a - the lower limit of integration.

$$I^{\sigma}K(t) = \frac{1-\sigma}{M(\sigma)}K(t) + \frac{\sigma}{\Gamma(\sigma)M(\sigma)}\int_a^t K(\tau)(t-\tau)^{\sigma-1}d\tau \tag{2}$$

Formula (2) define fractional integral of Atangana-Baleanu. This integral can be approximated using Euler method, which is described in [3], the approximation at $t = t_n$ as follows:

$$I^{\sigma}K(t) = \frac{1-\sigma}{M(\sigma)}K(t) + \frac{\sigma h^{\sigma}}{M(\sigma)\Gamma(\sigma+1)}\sum_{m=0}^{n-1}\theta_{\mathrm{n.m}}K(t_m), \tag{3}$$

where $\theta_{\mathrm{n.m}}$:

$$\theta_{\mathrm{n.m}} = (n-m)^{\sigma} - (n-m-1)^{\sigma} \tag{4}$$

Now we can rewrite formula (3) without sign of sum:

$$I_{Eu}^{\sigma}K(t) = \frac{1-\sigma}{M(\sigma)}K(t_n) + \frac{\sigma h^{\sigma}}{M(\sigma)\Gamma(\sigma+1)}K(t_{n-1}) + \frac{\sigma h^{\sigma}(2^{\sigma}-1)}{M(\sigma)\Gamma(\sigma+1)}K(t_{n-2})... \tag{5}$$

From formula (5) fractional differential coefficients can be extracted:

$$C_0 = \frac{1-\sigma}{M(\sigma)}$$
$$C_1 = \frac{\sigma h^{\sigma}}{M(\sigma)\Gamma(\sigma+1)}$$
$$C_2 = \frac{\sigma h^{\sigma}(2^{\sigma}-1)}{M(\sigma)\Gamma(\sigma+1)} \tag{5}$$

The noise reduction effect is achieved by processing the noisy image using convolution operation with a mask (see Fig.1) based on fractal coefficients C_0 … C_n.

C_2	0	C_2	0	C_2
0	C_1	C_1	C_1	0
C_2	C_1	$8C_0$	C_1	C_2
0	C_1	C_1	C_1	0
C_2	0	C_2	0	C_2

Figure 1. Denoise mask for eight directions:

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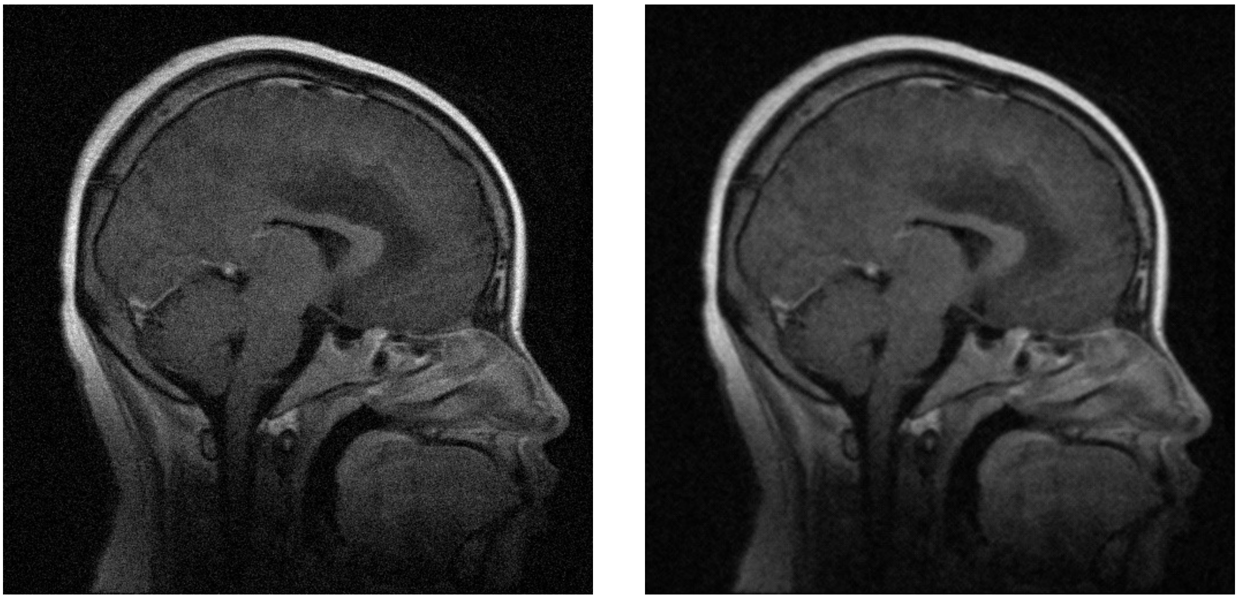
Fig. 1 illustrates the mask corresponding to eight symmetric directions. This mask allows calculating the fractional differential in all eight orientations. Such calculations ensuring consistent processing results regardless of the texture orientation within the image.



a) b)

Figure 2: (a) noisy MRI image, (b) denoised MRI image, using the mask from Fig. 1

At Fig.2. image with noise (a) and denoised image (b) displayed. The image obtained using the algorithm based on fractal differential operators with a sigma coefficient of 0.7 shows a noticeable reduction in visible noise, while textures and tissues have been well preserved.

Conclusion. The Atangana-Baleanu fractional operator effectively enhances MRI images by reducing noise, as confirmed through visual assessments. The results indicate that this algorithm demonstrating the advantages of fractional operators in improving image quality. This study highlights the potential of fractional operators for enhancing medical imaging and supports further exploration in this field.

References

[1] Behzad Ghanbari, Abdon Atangana, A new application of fractional Atangana–Baleanu derivatives: Designing ABC-fractional masks in image processing, Physica A: Statistical Mechanics and its Applications, Volume 542, 2020, 123516, ISSN 0378-4371, https://doi.org/10.1016/j.physa.2019.123516.

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[2] Ghanbari, Behzad & Atangana, Abdon. (2020). Some new edge detecting techniques based on fractional derivatives with non-local and non-singular kernels. Advances in Difference Equations. 2020. 10.1186/s13662-020-02890-9
[3] Li, C., Zeng, F.: The finite difference methods for fractional ordinary differential equations. Numer. Funct. Anal. Optim.34(2), 149–179 (2013)

Зменшення шуму в медичних МРТ - зображеннях за допомогою дробового оператора Атангана-Балеану

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У цій роботі досліджено фрактальні диференціальні оператори та їх застосування в обробці медичних МРТ-зображень, зокрема оператор Атангана-Балеану. Розглянуто чисельні апроксимації цього оператора та описано обчислення коефіцієнтів для апроксимації. Використовуючи ці апроксимації, створено маски з орієнтацією за напрямками для зниження шуму зображень, які інтегровано в спеціальний алгоритм і програмне забезпечення для покращення якості зображень. Виконано візуальне та аналітичне порівняння зображень без шуму з оригінальними, а також результати порівняно з фрактальними та традиційними методами зниження шуму. Дослідження завершується висновками щодо ефективності фрактального оператора Атангана-Балеану для оброблення МРТ-зображень.

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